

# The Importance of Modeling in Computational Musicology

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École Polytechnique Fédérale de Lausanne

*Round-table on "Probability and Music" – 5th International Congress of Music and Mathematics (MusMat 2020) – Perspectives and Applications of Mathematics in Post-Tonal Theories («Homage to Jamary Oliveira»), December 8–12, Rio de Janeiro, Brazil [online]*

- I. Modeling in Computational Musicology
- II. Example 1: Chord distributions
- III. Example 2: Pitch-class distributions
- IV. Perspectives

# I. Modeling in Computational Musicology

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- the (probabilistic) **relations** between those objects

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(cf. Finkensiep, Neuwirth, and Rohrmeier, forthcoming)

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- $P(I \mid V) = p$

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Potential of model-based computational musicology

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1. complementing music theory
2. resolving ambiguities in terminology
3. empirically validating theoretical assumptions
4. asking entirely new questions

## II. Ex. 2: Chord distributions

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Studying chord distributions to approximate **stylistic traits**, e.g. in

- **Western Classical music** (Jacoby, Tishby, and Tymoczko, 2015; Moss, Neuwirth, Harasim, and Rohrmeier, 2019)
- **Rock** (Temperley, 2018; Temperley and de Clercq, 2013)
- **Pop** (Burgoyne, Wild, and Fujinaga, 2011; Mauch et al., 2007)
- **Jazz** (Shanahan and Broze, 2012)
- **Choro** (Moss, Souza, and Rohrmeier, 2020)
- ...

## II. Ex. 2: Chord distributions

Common (implicit) **assumptions**:  $n$ -gram or Markov models:

$$p(c_i \mid c_1, \dots, c_{i-1}) \approx p(c_i \mid c_{i-n+1}, \dots, c_{i-1})$$

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- unigram model: relative frequencies ( $n = 1$ )
- bigram model: transitions ( $n = 2$ )

## II. Ex. 2: Chord distributions

**Unigram model:** relation between chord **rank** and **frequency** often approximated with Zipf-Mandelbrot law:

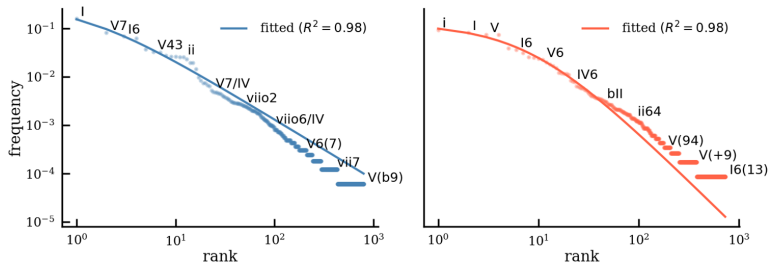
$$\hat{f}(r) = \frac{\alpha}{(\beta + r)^\gamma}$$



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**Figure 1:** Frequency-rank distribution of chords in Beethoven's string quartets (major: blue, minor: red; Moss, Neuwirth, Harasim, and Rohrmeier, 2019).

## II. Ex. 2: Chord distributions

### Bigram model: probabilities of chord transitions; conditional entropies

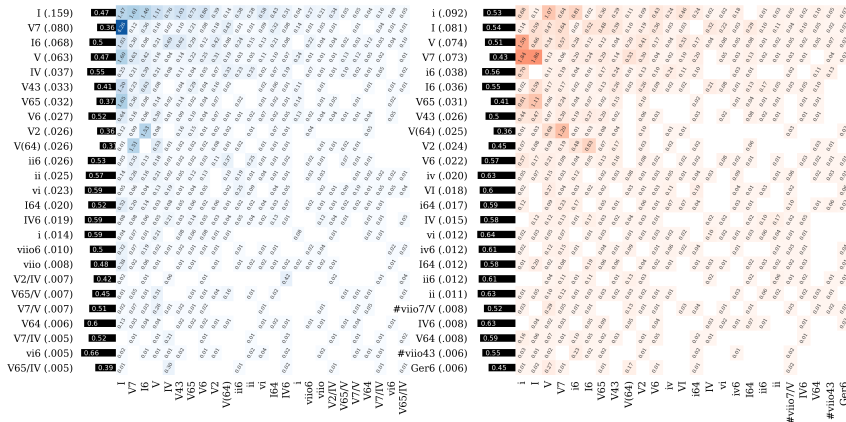


Figure 2: Transition probabilities in Beethoven's string quartets (major: blue; minor: red).

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1. Chord progressions seem to be **asymmetrical**, e.g.  $V2 \longrightarrow I6 \gg I6 \longrightarrow V2$

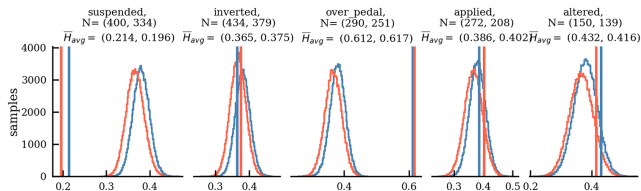
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Comparing average entropies of randomly sampled chords to those with certain features



**Figure 3:** Conditional entropies of chords with certain features (major: blue; minor: red).

### III. Ex. 2: Pitch-class distributions

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Theoretical (historical) models of **tonal space**

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## Theoretical (historical) models of tonal space

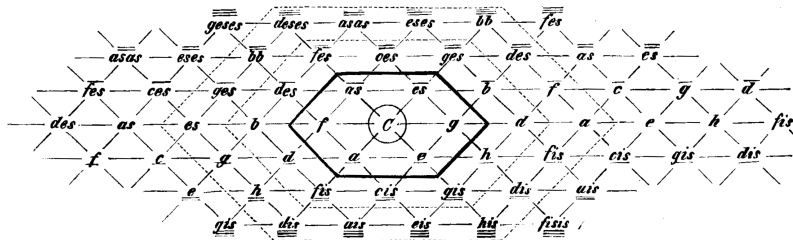


Figure 4: The *Tonnetz* (Hostinský, 1879).



### III. Ex. 2: Pitch-class distributions

Neo-Riemannian **triadic transformations** on the *Tonnetz* (Cohn, 1998)

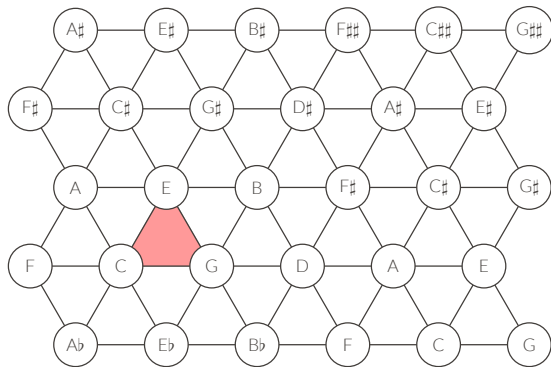


Figure 5: The *Tonnetz*.

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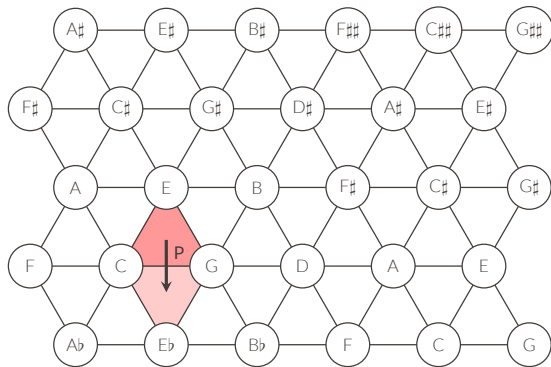


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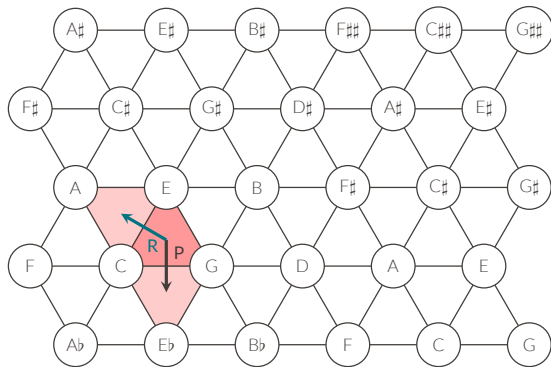


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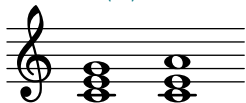
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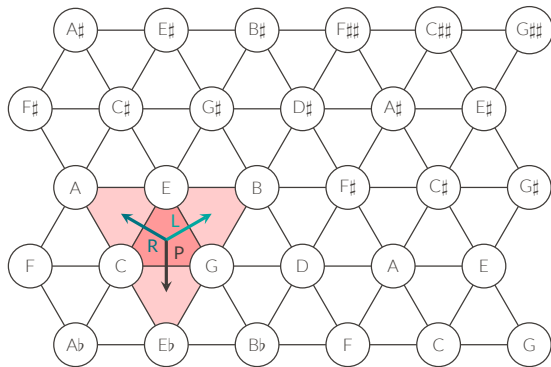
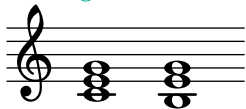


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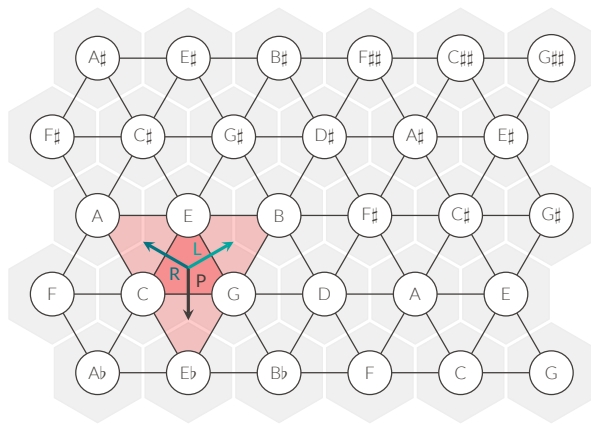
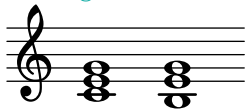


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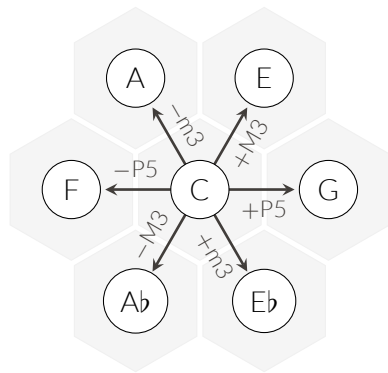


Figure 6: “Primary intervals” with respect to C.

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- (extended) **diatonic**:  $\pm P5$

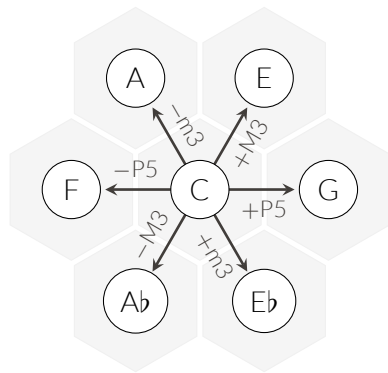


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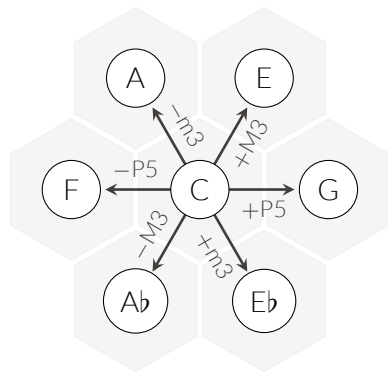
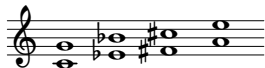


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- hexatonic**:  $\pm M3, \pm P5$

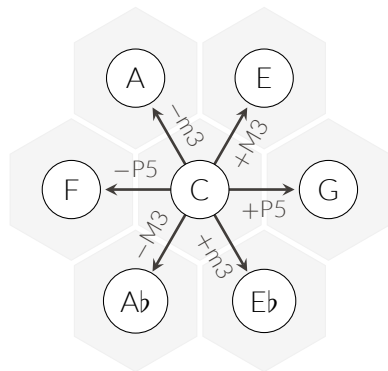
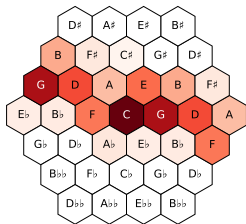


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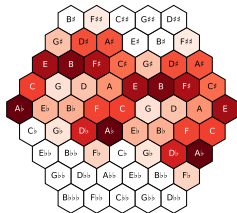
### III. Ex. 2: Pitch-class distributions

diatonic



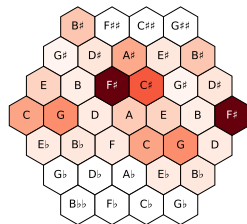
Bach, *Prelude in C major*,  
BWV 846 (1722).

hexatonic



Liszt, *Lugubre gondola I*,  
S. 200/1 (1882).

octatonic



Scriabin, *Prelude*,  
op. 74/2 (1914).

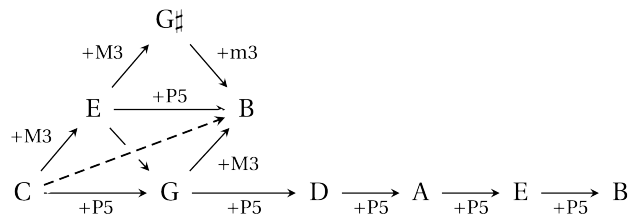
Plots generated with the *pitchplots* Python library (Moss, Loayza, and Rohrmeier, 2019)

### III. Ex. 2: Pitch-class distributions

Functional interpretation of interval relations on the Tonnetz:

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**Figure 7:** Different harmonic functions of B in relation to C.

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1. all notes are related to a tonal center

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The **Tonal Diffusion Model** – modeling assumptions:

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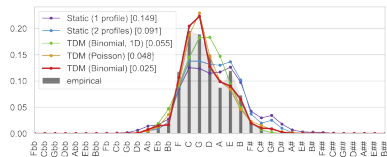
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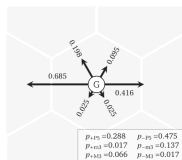
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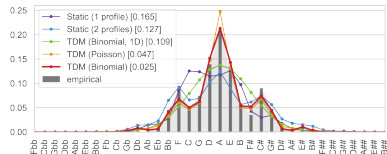
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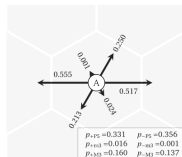
(a) Bach, C major Prelude.



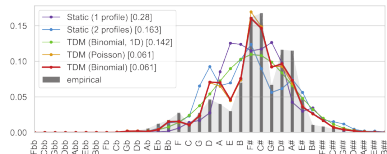
(a) Bach, C major Prelude  
( $\mu = 1.44, \sigma = 0.69$ ).



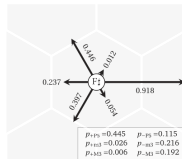
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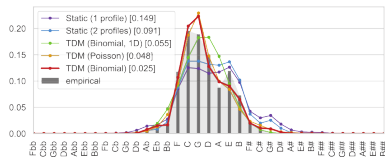


(c) Liszt, *Bénédiction de Dieu dans la Solitude*.

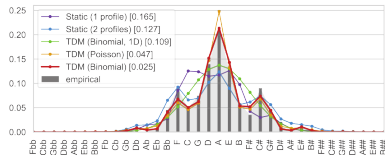


(c) Liszt, *Bénédiction de Dieu dans la Solitude* ( $\mu = 2.06, \sigma = 1.25$ ).

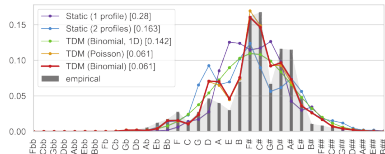
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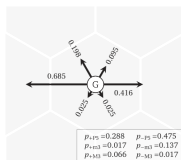
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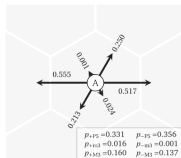
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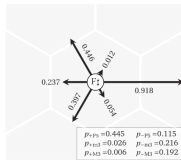
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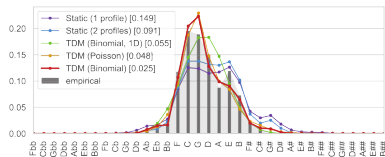
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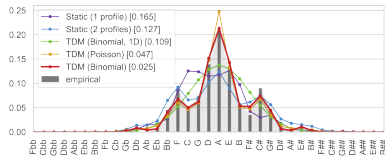
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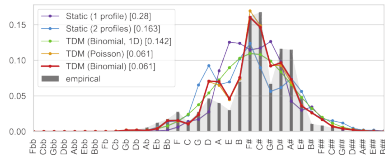
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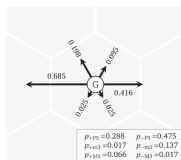
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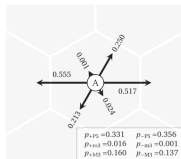
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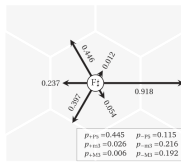
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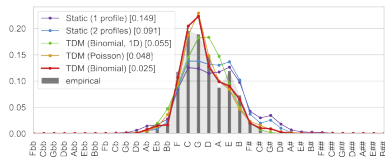
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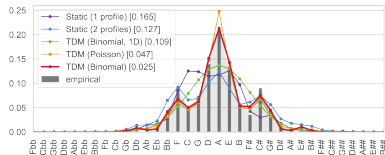
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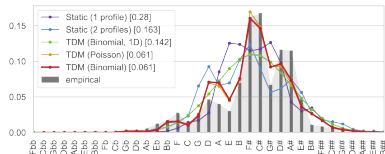
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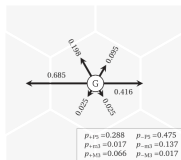
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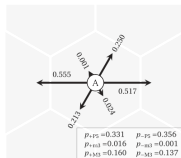
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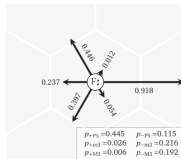
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2. **tonal center** is not identical to the tonic
3. **corpus-level comparison** shows that a 'line-of-fifths model' is sufficient for Bach but TDM is better for Beethoven and Liszt (see paper)

## IV. Perspectives

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4. probabilistic modeling & corpus-based approach can bridge between mathematical abstractions (e.g. pc sets) and musical data (e.g. pc distributions)
5. requires and facilitates reflection, critique, and interpretation

Thank you very much!

Slides: <http://www.fabian-moss.de>

# References

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